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Time Dependent reliability method for a Reinforced Concrete Coastal Structures due to Combine Overtopping and Structural Failure

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Abstract— This paper introduces a time-dependent reliability method to predict the probability of failures due to the increased wave overtopping and decreased structural capacity. Severe sea environmental conditions, sea storminess, global warming effects on sea levels, tidal effects etc has resulted in higher occurrence of wave overtopping and greater magnitude of hydrodynamic action on coastal defenses. The conditions has been aggravated for reinforced concrete coastal structures due to the seawater induced corrosion of reinforcing steel in concrete which reduces the load carrying capacity of the structure. A stochastic process is proposed to model the time variant and random nature of severe waves. Also proposed is a structural deterioration model to allow for seawater induced steel corrosion in concrete. It is clear in the example that the increasing wave heights, rising sea level and increased sea storminess reduce the safety and serviceability of coastal defenses.

Index Terms— Time dependent reliability method, stochastic process, Poisson process, Gaussian process, Wave overtopping, Structural failure

1. Introduction

THE Coastal structures, such as seawalls, breakwaters etc are essential components of sea defenses against flooding and coastline erosion and hence must be maintained at a high level of safety and serviceability. However, many coastal structures are currently under threat due to rising sea levels and increased sea storminess, which result in higher frequency of wave overtopping and greater magnitude of hydro-dynamic action on the structures. The situation has been aggravated for reinforced concrete (RC) coastal structures, primarily due to the seawater induced corrosion of the reinforcing steel in concrete which reduces the load bearing capacity of the structure. This combined effect will lead to inevitable reduction of the designed service life of coastal concrete structures, posing a higher risk to the public at large. The consequence of failures of coastal defences (however they fail) is catastrophic and the cost is substantial.

Coastal structures can fail in different modes which, technically, can be divided into two categories [1]: (1) the attainment of serviceability limit state, such as sliding or wave overtopping and (2) the attainment of ultimate limit state, such as toppling or rupture. As one of the most important failure modes for coastal structures, overtopping has been studied considerably in the past decades, with perhaps more emphasis on experimental and numerical investigations than on analytical ones (e.g., [14] [4] [2] [8],[3], [13]). Probabilistic risk assessment of wave overtopping has also been undertaken in several studies ([16], [7]) where the discharge volume is treated as a random variable based on classic probability theory. A review of the research literature reveals that, while considerable research has been undertaken on the modeling of wave overtopping (see above references), the time effect of the overtopping process has not been taken into account explicitly. This has overlooked the stochastic nature of the phenomenon.

In this paper, a time-dependent reliability method is introduced to predict the risk of failures of RC seawalls due to the increased wave overtopping and reduced structural capacity. A stochastic model of the Poisson renewal process is proposed to describe the time variant and random nature of severe waves. Also proposed is a structural deterioration model for RC flexural structures affected by steel corrosion in concrete. An example is provided to illustrate the proposed method whereby some of important factors that affect the risk are also studied.

2. Formulation of probability Assessment

In assessing the probability of failures for a coastal structure, a performance function should be established for the structure. In reliability theory, this function is expressed in the form of a limit state function as follows:

$$G(L,s,t) = L(t) - S(t) \quad (1)$$

Where, $L(t)$=random capacity for the action or its effect; $S(t)$=action (random load) or its effect; and t=time. With Eq. (1), the probability of failure for the structure $P_f(t)$ can be determined by,

$$P_f(t) = P_r[G(L,S,t) \leq 0] = P_r[S(t) \geq L(t)] \quad (2)$$

Where $Pr(\ )$ denotes the probability of an event. Eq. (2)

represents a typical crossing problem and can be dealt with using time-dependent reliability methods.

In this method, the probability of failure depends on the time that is expected to elapse before the first occurrence of the action process *S*(*t*) crossing an random capacity (the threshold) *L*(*t*) sometime during the service life of the structure (0, T_L). Equivalently, the probability of the first occurrence of such an excursion is the probability of failure ($P_f(t)$) during that time period. This is known as ―first passage probability ‖ and can be determined by ([12]),

$$P_f(t) = 1 - [1 - P_f(0)]e^{-\int_0^\infty v\, d\tau} \quad \text{------------- (3)}$$

Where $P_f(0)$ =probability of failure at time *t*=0 and v=mean rate for the action process *S*(*t*) to cross the threshold *L*(*t*). For Eq. (3) to be applied to risk assessment of coastal structures, two types of stochastic processes for *S*(*t*) are considered in this paper: (1) a discrete Poisson renewal process and (2) a continuous Gaussian process. A Poisson renewal process is a pulse process in which the occurrence time of the pulse, its duration, and its intensity are all treated as random variables ([15]). When the action process *S*(*t*) is modeled as a Poisson renewal process, the crossing rate (in Eq. (3) can be determined by ([9])

$$\lim_{\Delta t\to 0}\{\tfrac{1}{\Delta t}[P(crossing\ in\ \Delta t]\lambda\} = \lim_{\Delta t\to 0}[\tfrac{1}{\Delta t}(P\{[S(t)\le L(t)]\cap[S(t+\Delta t)>L(t)]\}\lambda] = F_s[L(t)]\{1-F_s[L(t)]\}\lambda \quad \text{---------- (4)}$$

Where ( ) probability distribution function for the action *S*(*t*) is modeled as a continuous Gaussian process, the crossing rate can be determined from the Rice formula (Melchers 1999)

$$v(t) = \int_{\dot L}^{\infty} (\dot S - \dot L) f_{ss}(L,\dot S)\, d\dot S \quad \text{----- (5)}$$

Where v =crossing rate of the action process *S*(*t*) relative to the threshold *L*(*t*); $\dot L$ =slope of *L*(*t*) with respect to time; (*t*)=time derivative process of *S*(*t*); and $f_{ss}(\ )$=joint probability density function for *S* and . An analytical solution to Eq. (5) has been derived for a deterministic threshold *L* in [11] as follows:

$$v = v_L = \frac{\sigma_{\dot S|S}}{\sigma_S}\phi\frac{(L-\mu_S)}{\sigma_S}\left[\phi\left(\frac{\dot L-\mu_{\dot S|S}}{\sigma_{\dot S|S}}\right) - \frac{\dot L-\mu_{\dot S|S}}{\sigma_{\dot S|S}}\Phi\left(-\frac{\dot L-\mu_{\dot S|S}}{\sigma_{\dot S|S}}\right)\right] \quad \text{------- (6)}$$

Where, v_L denotes the crossing rate relative to the threshold *L*; $\phi(\ )$ and $\Phi(\ )$=standard Gaussian density and cumulative distribution functions, respectively; and denote the mean and standard deviation (SD) of *S* and $\dot S$, represented by subscripts; and "|" ‖denotes the condition. For a given Gaussian stochastic process with mean function $\mu_s(t)$ and auto-covariance function $C_{\varphi\varphi}(t_i, t_j)$ all variables in Eq. (6) can be determined, based on the theory of stochastic processes as detailed in [15] and also[11].

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When a structure can fail in multiple modes, the probability of structural failure can be determined using the methods of system reliability [12]. There are basically two systems in the theory of system reliability. One is known as the series system in which the occurrence of one failure mode constitutes the failure of the system. The other is known as the parallel system in which the system fails only when all failure modes occur. As discussed in the Introduction, two failure modes are important for RC vertical seawalls: (1) wave overtopping and (2) structural failure (rupture). These two failure modes are considered in the paper. Obviously, the occurrence of either failure mode will constitute the failure of the seawall. Therefore a series system is appropriate for its risk assessment. According to the theory of system reliability [12], the probability of failure for a series system ($P_{f,s}$) can be estimated by

$$max[P_{f,i}] \le P_{f,s} \le 1 - \prod_{i=1}^{n}[1 - P_{f,i}] \quad \text{-------- (7)}$$

Where $P_{f,i}$ =probability of failure due to the *i* th failure mode [determined by Eq. (3)] and *n*=number of failure modes considered. Herein, *n*=2.

2.1. Probability of Overtopping

Overtopping occurs when the waves of a storm run-up a protecting structure, e.g., seawalls or breakwaters, and pass over the crest of the structure ([3]). An analysis of the performance of a variety of coastal structures shows that the frequencies, discharge volumes, and velocities of waves are key factors that affect overtopping. It also shows that, for normal seawalls with high freeboard, only a small number of overtopping events can occur during the designed lifetime of the structure. These are generally caused by large unbroken waves (exceeding a threshold), which are referred to as severe waves in this paper. Examining the characteristics of these severe waves ([3], [16]), the following features can be identified:

1. Randomness of occurrence time. Severe waves come about from time to time during the design life of seawalls

and at a given time, it is not certain whether a severe wave occurs or not;

2. The randomness of intensity. When a severe wave occurs, its discharge volume is uncertain; and3. Randomness of duration. When a severe wave occurs, it is not known how long it would last.

Based on these characteristics, it is appropriate to model the occurrence of the severe waves as a Poisson renewal process, in which each pulse represents a severe wave as measured by a discharge volume *V* (Fig. 1). Generally, independence is assumed between the pulse intensities and the duration in each occurrence of the pulses; independence is also assumed from one occurrence to another. The Poisson process of severe waves can be described by a mean occurrence rate of waves λ_V, a duration *d*, and the magnitude of discharge volume *V*. Mathematically the Poisson process of severe waves can be expressed as (e.g., [10])

$$V(\lambda_V d, V_i, t, t_i) = \begin{cases} V & \text{when it occurs} \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

Where, *i* refers to the *i* th severe wave and *t* denotes time (in years). In assessing the risk of overtopping by severe waves, a performance criterion can be established, according to Eq. (1), as follows:

$$G(v_P, V, t) = v_P(t) - V(t) \qquad (9)$$

Where, v_P =acceptable (permissible) limit for the discharge volume during the service life (0, T_L) of the seawall. Since in most practical applications, the acceptable limit for discharge volume v_P is a constant, such as prescribed in design codes and standards (e.g., BS6348), the risk of wave overtopping can be determined, for a given height of severe wave *Hs*, from Eqs. (3) and (4), with *V* replacing *S* and v_P replacing *L* (mathematically an up-crossing problem), as follows:

$$P_{f,o}(t|H_s) = 1 - [1 - P_f(0)].\,exp\left\{-\int_0^t F_V(v_P)[1 - F_V(v_P)]\lambda_V d\tau\right\} \qquad (10)$$

Where $P_{f,o}(t|H_s)$= conditional probability of wave overtopping for a given wave height *Hs*.

The number of waves overtopping a vertical seawall *Ns* over a period of time, e.g., [0, T_L] can be estimated approximately by ([3])

$$\frac{N_s}{N_w} = \left[-\left(\frac{1}{0.91}\right)\left(\frac{R_c}{H_s}\right)^2\right] \qquad (11a)$$

$$\frac{N_s}{N_w} = 0.031 R_h^{-0.99} \qquad (11b)$$

Where N_w =total number of waves in the period; R_c=freeboard (the height of the crest) of the seawall above still water level (*h*, as shown in Fig. 2); R_h =dimensionless freeboard determined by

$$R_h = (R_c/H_s)h^* \qquad (11)$$

And h^*= wave breaking parameter determined by

$$h^* = \frac{h}{H_s}\left(\frac{2\pi h}{gT_m^2}\right) \qquad (12)$$

In Eq. (12), *h* = water level (depth) at the toe of the structure (in meters); *g* = acceleration due to gravity in (m/s^2); and T_m= mean wave period at the toe of the structure (in seconds). From Eqs. λ_V (11*a*), (11*b*), and (12), the mean occurrence rate of severe waves, which represents the number of severe waves per unit time, can be obtained as follows:

$$\lambda_V = \frac{N_s}{N_w T_m} \qquad (13)$$

As the height of severe waves, H_s = random variable, the (unconditional) probability of wave overtopping can be determined as follows:

$$P_{f,o}(t) = \int_0^\infty P_{f,o}(t|H_s)\, f_H(H_s)\, dH_s \qquad (14)$$

Where, $f_H(H_s)$= probability density function for *Hs.*

2.2. Probability of Structural Failure

Severe waves not only incur overtopping but also exert higher (than service load) wave pressure on defending structures. Together with seawater-induced reduction of structural capacity, the risk of structural failure (i.e., rupture) increases. In assessing the risk of the structural failure, a performance criterion can be established, according to Eq. (1), as follows:

$$G(M_c, M_h, t) = M_c(t) - M_h(t) \qquad (15)$$

Where, $M_c(t)$ =flexural capacity of the seawall and $M_h(t)$= overturning moment caused by wave action (hydrodynamic Pressure).

In this paper, the model proposed by Goda ([6]), which has been widely used in practice, is employed to estimate the overturning moment $M_h(t)$ as follows:

$$M_h = \int_0^h y Z_B(y)dy + \int_h^{R_c} y Z_A(y)dy \qquad (16)$$

Where, $Z_B(y)$ and $Z_A(y)$ wave pressure on the seawall at height *y* from the toe of the seawall. In determining the flexural capacity of the seawall the reduction of the structural capacity due to seawater-induced corrosion should be considered. To model the corrosion-induced capacity reduction, a deterioration function can be introduced as follows:

$$M_c(t) = \varphi(t) M_0 \qquad (17)$$

Where, $\varphi(t)$ deterioration function and M_0 =original structural capacity determined in the design.

Since both the corrosion process and structural deterioration are not only random but also time variant, it is well

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justified to model the deterioration function as a stochastic process with a mean function $\mu_\varphi(t)$ and an auto-covariance function $C_{\varphi\varphi}(t_i, t_j)$ in a general form of

$$\mu_\varphi(t) = \varphi_0 exp(-\zeta t) \qquad \text{----------- (18)}$$

$$C_{\varphi\varphi}(t_i, t_j) = \rho_\varphi \sigma_\varphi(t_j) \sigma_\varphi(t_j) \qquad \text{----------- (19)}$$

Where, φ_0 accounts for initial defects and ξ=coefficient representing the rate of structural deterioration. It allows for the effects of such factors as degree of corrosion, concrete quality, and structural detailing. In Eq. (18*b*), $\sigma(t)$ standard deviation function and ρ_φ = autocorrelation coefficient for deterioration function $\varphi(t)$ (a stochastic process) between two points in time t_i and t_j with the limit state function of Eq. (15) and the models for both and available, the risk of structural failure due to the combined effect of the increased wave action and structural deterioration can be determined, for a given wave height *Hs* and $M_h(t)$ hence, from Eqs. (3) and (6) with *Mc* replacing *S* and M_h replacing *L* (mathematically a down-crossing problem) by,

$$P_{f,s}(t|M_h) = 1\text{-}[1 - P_f(0)].\,exp\left\{-\int_0^t \frac{\sigma_{M_c|M_c}}{\sigma_{M_c}} \phi\left(\frac{M_h-\mu_{M_c}}{\sigma_{M_c}}\right) \times \left[\phi\left(\frac{\mu_{M_c|M_c}}{\sigma_{M_c|M_c}}\right) + \frac{\mu_{M_c|M_c}}{\sigma_{M_c|M_c}} \Phi\left(\frac{\mu_{M_c|M_c}}{\sigma_{M_c|M_c}}\right)\right] d\tau\right\} \qquad \text{---------- (20)}$$

Where, $P_{f,s}(t|M_h)$ = conditional probability of structural failure for a given overturning moment M_h. Since the wave height *Hs* is modeled as a random variable as in the case of overtopping, the resultant overturning moment M_h is also a random variable. Denoting $f_M(M_h)$ as the probability density function for M_h, the (unconditional) probability of structural failure due to the increased wave action and structural deterioration can be determined by,

$$P_{f,s}(t) = \int_0^\infty P_{f,s}(t|M_h)\, f_M(M_h)\, dM_h \qquad \text{------------ (21)}$$

3. WORKED EXAMPLE

3.1. Risk of Overtopping

This worked example demonstrates the basic application of the proposed method to a RC vertical seawall. It can also serve as a guide as to what data are needed in the hazard assessment of coastal structures. All the information for basic design parameters and variables of the seawall and wave conditions is provided in Table 1.

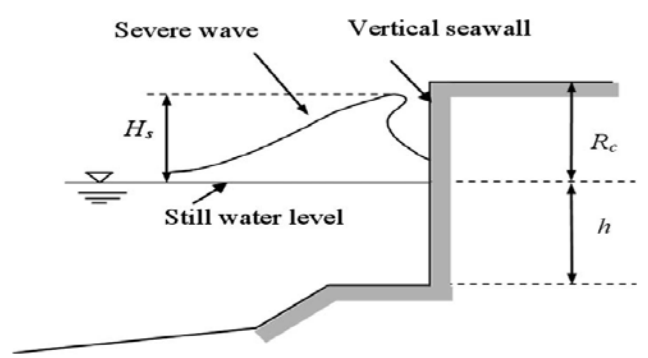

Fig. 1 Illustration of a severe wave at a vertical seawall

TABLE 1: BASIC VARIABLES AND PARAMETERS

Description	Symbol	Value
Freeboard	*Rc*	5.0 m
Still water level at the toe	*h*	7.0 m
Mean wave period	*Tm*	6 s
Acceptable limit of overtopping	v_p	0.2 m3 /s·m
Wavelength	*L*	20 m
Water depth from 5*HD* in front of the base of seawall	h_b	12 m
Water depth in front of the base of seawall	*d*	10 m
Angle of incidence of the wave attack	β	0

For the risk assessment of overtopping, the height of severe waves *Hs* is assumed to follow the Weibull distribution with the mean value of 1.5 m and the standard derivation of 0.6 m ([8]). Together with Table 1, other variables used in calculation can be determined accordingly. For example, for a given value of *Hs* and values of other parameters given in Table 1, the wave breaking parameter can be obtained from Eq. (12) and subsequently the number of overtopping waves from Eq. (11).

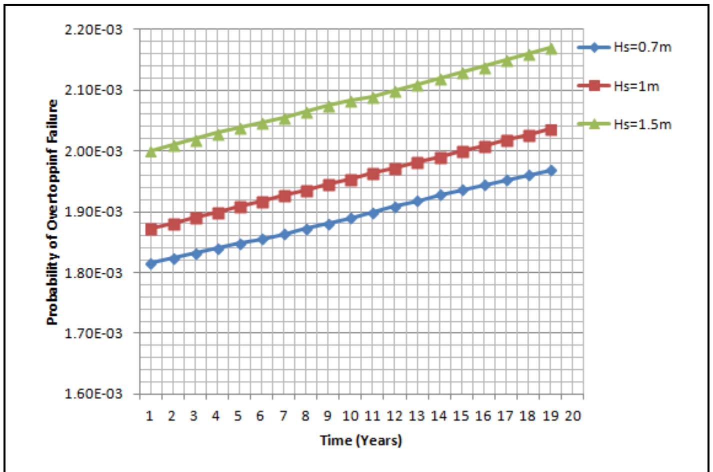

Fig. 2 Risk of overtopping versus time for different heights of severe

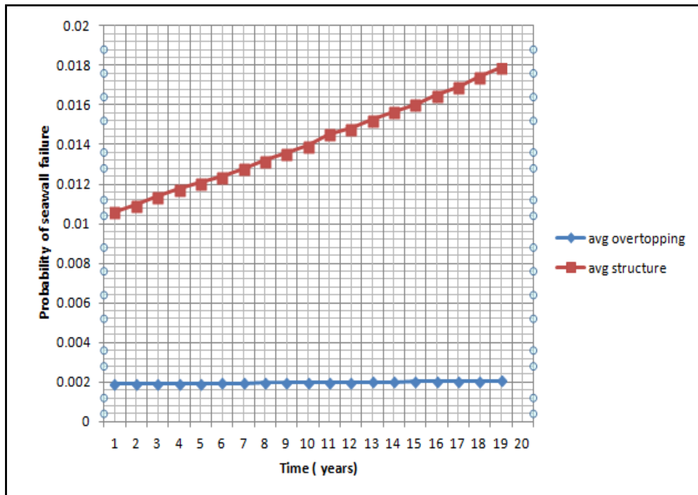

Fig. 4 Risk of seawall failure versus time due to different modes

The mean occurrence rate of severe waves can then be determined from Eq. (13). As for the discharge volume of overtopping waves, it has been found to follow Weibull distribution (e.g., [8], [4], [13]).

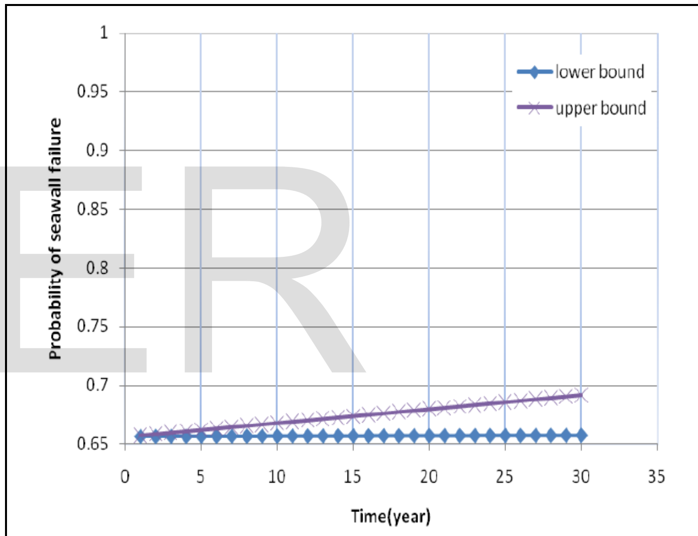

Fig. 5 Combined risk of wave overtopping and structural failure versus time

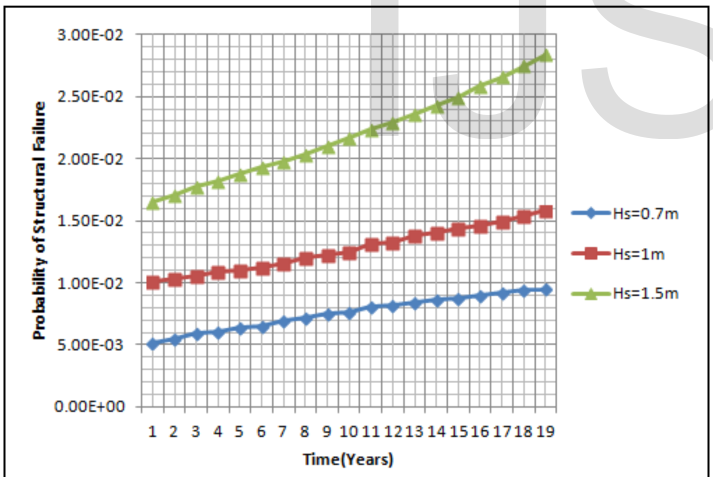

Fig. 3 Risk of structural failure versus time for different heights of severe wave, i.e., overturning moment

An effective measure to reduce the probability of wave overtopping is to increase the height of seawall. This makes sense both theoretically and practically. More importantly, the effect of height of seawall on the risk in a long run (e.g., from 50 to 100 years) is greater than that in a relative short term (e.g., from 10 to 20 years). As the cost of increasing the height of seawalls is high, a risk-cost optimized approach is important to achieve the cost effectiveness in the design and management of seawalls. Fig. 2 shows that the probability of overtopping is highly related to the height of severe waves (*Hs*). It can also be seen that the effect of the average height of severe wave on the probability of overtopping increases over time.

3.2. Risk of Structural Failure

For the given wave height *Hs*, the wave action induced overturning moment can be determined from Eq. (16) and is also assumed to follow the Weibull distribution. As for structural (residual) capacity, the proposed deterioration function in Eq. (18) can be adjustd against the data which is produced from the tests on corrosion-induced structural deterioration [10]. The probability of the structural failure due to the increased wave action and structural deterioration can be estimated using Eqs. (19) and (20).

The results are summarized in the Fig. 3, the probability of structural failure increases with the increase of the height of severe waves and hence the overturning moment. This indicates the impact of increased wave actions on structural safety. To allow for the increased wave action, the structural capacity needs to be increased or strengthened. The correlation of a stochastic process between two time points is usually not readily available. The results are reliable with the research experience in stochastic processes (e.g.[11]).

The probabilities of wave overtopping and structural failure are shown in Fig. 4 individually. As can be seen the probability of structural failure increases faster than the risk of wave overtopping over time, indicating the interactive effect of the increased wave action (i.e., overturning moment) and structural deterioration. Finally, the combined probability of wave overtopping and structural failure for the seawall can be estimated from Eq. (7) and the results are shown in Fig. 5.

4 CONCLUSION

A time-dependent reliability method has been introduced in the paper to predict the probability of failures of RC vertical seawalls due to the increased wave overtopping and reduced structural capacity. In this method, the time variant and random nature of severe waves is modeled by a Poisson process. A structural deterioration model is proposed to tolerate for the reduction of structural capacity caused by steel corrosion in concrete. A worked example has been provided to demonstrate the basic application of the proposed method. Results from the example suggest that the average height of severe waves, the seawater level, the permissible discharge volume and the height of seawall are important factors that affect the probability of coastal structural failures. Results also suggest that the rising sea levels and increased sea storminess decrease the safety and serviceability of coastal structures and the probability of structural failure increases faster than the probability of wave overtopping for RC vertical seawalls. It can be concluded that the method presented in the paper can provide some useful information for coastal defenses.

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